Analytical model for deep-drawing of a cylindrical cup

R.A. Nazarov,[1,2] Z. Ayadi,[2] S.A. Nikulin[1]

[1]*Moscow Institute of Steel and Alloys (technological university), 119049, Moscow, Leninskij avenue, 4, Russia*

[2]*National Polytechnical Institute of Lotraine, 54010, Nancy,Bastien Lepage street, 6, France*

romannazarov@yahoo.com

Introduction

Mathematical description of the processes of sheet metal forming is very complicated because of varying loading history and complex stress state in each point of material. The parameters for behaviour law introduced in the mathematical models of sheet metal forming are normally taken from a simplest mechanical test such as, for example, uniaxial traction test. Lately some new tests were developed which are closed to reel forming processes. Deep-drawing of a cylindrical cup occupies a particular place between these tests since it allows to study material hardening, conditions of friction, springback, wrinkling, plastic flow instability, fracture and some others effects at the same time [1,2].

There are two stages of deep-drawing: during the first one the blank holder descends on a circular blank of diameter $2R_{ext0}$ and thickness h_0 and press it with force F_{bh}; the process of forming takes place at the second stage when the cylindrical punch descends on the blank and draw it in the hole of matrix (Fig. 1). One can control the metal flow in matrix by modifying the blank-holder force that changes the friction force (coefficient of friction μ).

Usually the process keeps on up to the moment of full drawing of the cup. But this is not always possible because of the fracture of the cup generally in the zone of cylindrical wall. It is obvious that the force of material resistance to drawing depends on the diameter of the initial blank against the punch diameter (R_{ext0} / R_p) – for bigger value of this ratio the force of resistance is bigger. Therefore there is a limiting drawing ratio (LDR) for which full cup drawing is still possible. In one of the first analytical models [1] of this process Hill predicted the upper and the lower drawing limits for isotropic non-hardening material ($\sqrt{e} = 1{,}65$ and $e = 2.72$ respectively). Later models [3-7] improved this approach by considering the current geometry of the process (Fig.1) in order to determine the strain rate field and then the stress field and the forces. In each of these models some simplifications were applied, in particular, the hypothesis of cup thickness unchanged during all the process.

The aim of this work is the determination of the adequacy of these models, theirs limitations and the development of a new model to overcome these limitations.

Analysis of deep-drawing models

The current geometry of process when the punch has penetrated in matrix at a distance d is presented in Fig.1. Flange has an exterior diameter (changing in the time) equal to $2R_{ext}$, and interior diameter (which is unchanged) equal to $2R_{\text{int}}$. The radius of matrix and punch rims are r_p and r_m respectively.

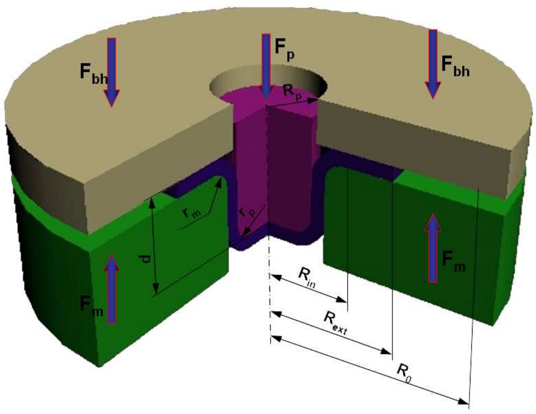

Fig.1. Current geometry of cylindrical deep-drawing

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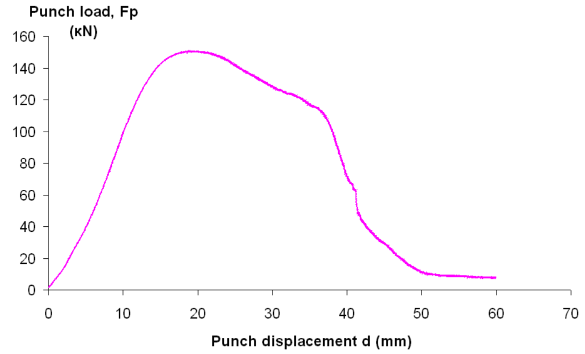

Fig.2: Punch load as a function of the punch displacement

Force F_p, with which the punch acts on the blank increase at the beginning of process because of material hardening in flange and it decreases at the end of process as a result of flange reduction (Fig.2). This curve was obtained for DP600 steel chemical parameters for which are shown in Table 1 and parameters of experience are presented in Table 2. The initial circular blank of a diameter equal to 0.141 m was pressed by a blank holder with force 5000N and then was drawn in order to obtain a cup. During this experiment two parameters were measured: the force of punch by dynamometer installed on the punch and the punch displacement.

Table 1. Chemical composition of DP600 steel

Steel	Si	Mn	Cr	Ni	Ti	Cu	V	Mo	[O]	[N]	[C]	[S]
DP600	0,35	1,13	0,68	0,06	0	0,09	0	0	0,0068	0,0044	0,23	0

Table 2: Experimental parameters

Steel	Elastic limit σ_0, MPa	Coefficient of anisotropy $\bar{r}$	Coefficient of strain-hardening K, MPa
P600	155	2,1	610
Exponent of strain-hardening, n	Sheet thickness h_0, mm	Radius of punch R_p, mm	Radius of matrix rim r_m, mm
0,263	1	30,75	8,5

The curve $F_p(d)$ has a maximum $F_{p\max}$ corresponding to a maximal force of flange resistance to drawing.

There are several constituents in the punch force:

$$F_p = F_{contr}^{fl} + F_{fr}^{fl} + F_{contr}^{rim} + F_{fr}^{rim} + F_{bend} + F_{unbend} + F_{fr}^{s-w} \quad (1)$$

where F_{contr}^{fl} is the force due to material hardening in flange; F_{fr}^{fl} is the friction force in flange; F_{contr}^{rim} is the force due to material hardening in matrix rim; F_{fr}^{rim} is the friction force in matrix rim; F_{bend} is the bending force at the beginning of matrix rim and F_{unbend} is the unbending force at the end of matrix rim and F_{fr}^{s-w} is the friction force in side wall.

In the most of models the friction force in side wall F_{fr}^{s-w} is not taken into account as it is hardly can be evaluated and generally there is not a contact between cup wall and matrix.

The bending force F_{bend} can be determined by the equation initially proposed by Stoughton [8]:

$$F_{bend} = 2\pi R_{\text{int}} \int_{h_0/2}^{h0} dy \int_{\varepsilon_0}^{\varepsilon(y)} \sigma \, d\varepsilon \quad (2)$$

The force of contraction in matrix rim F_{contr}^{rim} is proportional to the radius of matrix rim (r_m). It can be determined by means of the following equation:

$$F_{contr}^{rim} = 2\pi R_{\text{int}} h \left(\sqrt{\frac{2(\bar{r}+1)}{2\bar{r}+1}} \right)^{n+1} K \int_{R_{in}-r_m-h0/2}^{R_{in}} \left(\ln \frac{r_0}{r} \right)^n \frac{dr}{r} \quad (3)$$

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The friction force in the matrix rim F_{fr}^{rim} is proportional to the portion of punch force, that was created in flange F^{fl}:

$$F_{fr}^{rim} = \left(F_{contr}^{fl} + F_{fr}^{fl}\right)\left(e^{\mu\alpha} - 1\right) \quad (4)$$

For the given force of flange resistance F_{fr}^{rim} to the deep-drawing this force depends only on friction coefficient μ and cover angle α that is normally equal to $\pi / 2$.

In the early models [1-4] the contribution of matrix rim and side wall to the punch load was neglected. This could be the cause of the loss of 20-30% of exact solution.

There are many ways to determine the friction force in flange F_{fr}^{fl} [3,4,5]. All of these methods are semi-empirical.

For contraction force in flange F_{contr}^{fl} in the case of non-hardening material Hill [1] proposed the following expression, based on the hypothesis of unchanged sheet thickness:

$$F_{cont}^{fl} = \sigma_0 \left[\frac{2(\bar{r}+1)}{2\bar{r}+1}\right]^{1/2} \ln\frac{R_{ext}}{R_{\text{int}}} \cdot 2\pi h R_{\text{int}} \quad (5)$$

Well, all models described above have some strong hypotheses which do not correspond to real process. Two of the most important hypotheses concern the thickness change and friction effect.

In our model a method of the most precise solution for contraction force F_{cont}^{fl} and friction force F_{fr}^{fl} in flange is proposed as they have the biggest contribution in punch load.

Description of the new model

In contrast to the described model [1, 3-7] that consider unchanged sheet thickness and facilitate a lot the calculations (a 3D problem becomes a 2D problem) in this work we propose to take into calculus an identical in each point of the flange blank-holder pressure. This situation occurs in the case of hydraulic blank-holder [9] or soft blank-holder [10].

There are 6 principal equations that were used in our model:

- Equilibrium equation in the cylindrical coordinates (shear stresses were neglected):

$$\frac{d\sigma_r}{dr} = \frac{\sigma_\theta - \sigma_r}{r} - 2\frac{\mu\sigma_z}{h} \quad (6)$$

- Ludwig hardening law: $\bar{\sigma} = \sigma_0 + k\bar{\varepsilon}^n$ (7)
- Hill 48 effective stress for anisotropic materials:

$$\bar{\sigma} = \frac{1}{\sqrt{\bar{r}+1}}\left[\bar{r}(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2\right]^{1/2} \quad (8)$$

- plastic flow laws

$$\frac{d\varepsilon_r}{\bar{r}(\sigma_r - \sigma_\theta) + (\sigma_r - \sigma_z)} = \frac{d\varepsilon_\theta}{\bar{r}(\sigma_\theta - \sigma_r) + (\sigma_\theta - \sigma_z)} = \frac{d\varepsilon_r}{(\sigma_z - \sigma_r) + (\sigma_z - \sigma_\theta)} = \frac{d\bar{\varepsilon}}{(1+\bar{r})\bar{\sigma}} \quad (9)$$

- Hill 48 effective strain: $d\bar{\varepsilon} = \frac{\bar{r}+1}{\sqrt{2(\bar{r}^2+\bar{r}+1)}}\sqrt{d\varepsilon_r^2 + d\varepsilon_\theta^2 + d\varepsilon_z^2}$ (10)
- hypothesis of plastic incompressibility: $d\varepsilon_r + d\varepsilon_\theta + d\varepsilon_z = 0$ (11)

Solving these equations with the known limiting conditions we obtained the solution for the deep-drawing of cylindrical cup. The program of modelling is written in FORTRAN 90.

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Comparison of the different models with experiment

The results of the calculus by new model were compared with experimental results with the parameters presented in Table 2.

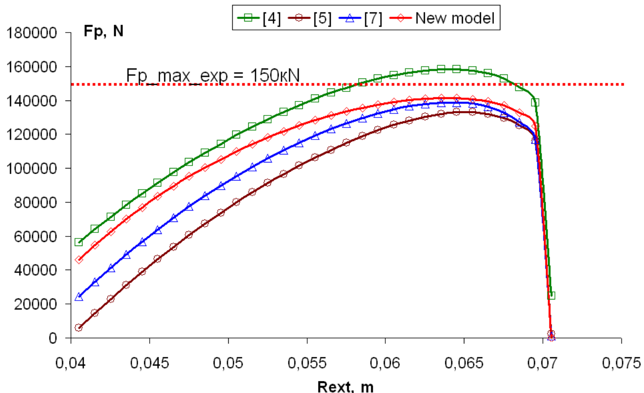

Fig.3: Punch load as a function of the exterior flange radius

In Fig. 3 there is graph of punch load F_p as a function of exterior flange radius R_{ext} calculated by 4 different models (Leu [4], Verma [5], Schedin [7] and new model) and also the maximal value of punch load from the experiment (fig. 2).

As one can see from fig. 3, new model gives the best according to experiment value of $F_{p\max}$. Model [4] overestimates $F_{p\max}$ as compared with experience and three others models underestimate its value.

Conclusion

The most important hypothesis used in analytical models of deep-drawing processes concern the thickness change during the process and modelling of friction.

Most of analytical models consider the unchanged thickness of the metal sheet that complicates the modelling of friction. The proposed new model consider an identical in each point of flange blank-holder pressure that allows the determination of friction force in more natural way by using a standard Coulomb law.

The results obtained using new model are in the best agreement with the experimental ones as compared with others analytical models.

Combination of the known models [1,3-7] and the new one allows an improved calculation of the maximal punch load and subsequently the LDR determination.

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